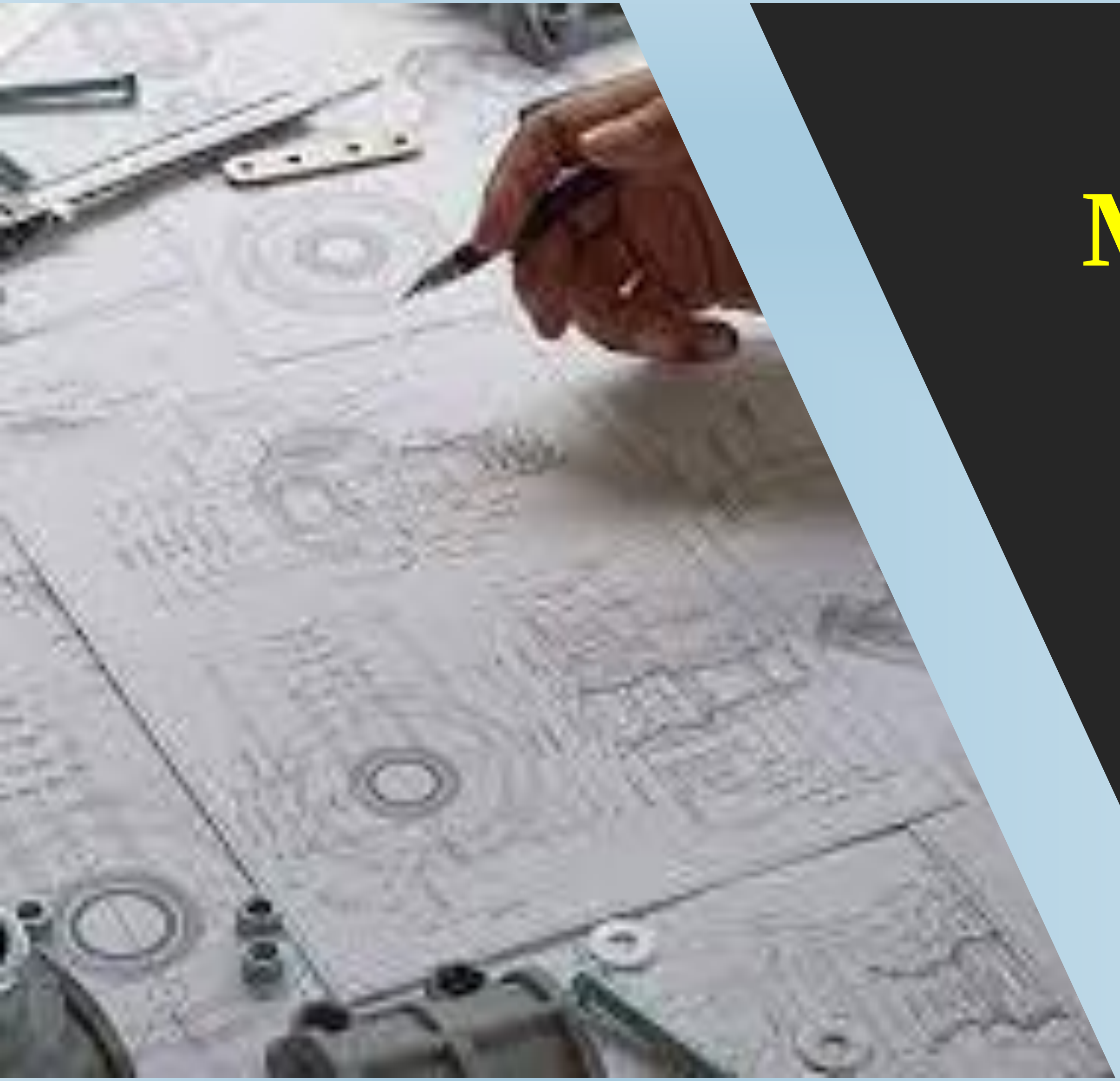

MECHANICS

Lecture No.5

Equilibrium

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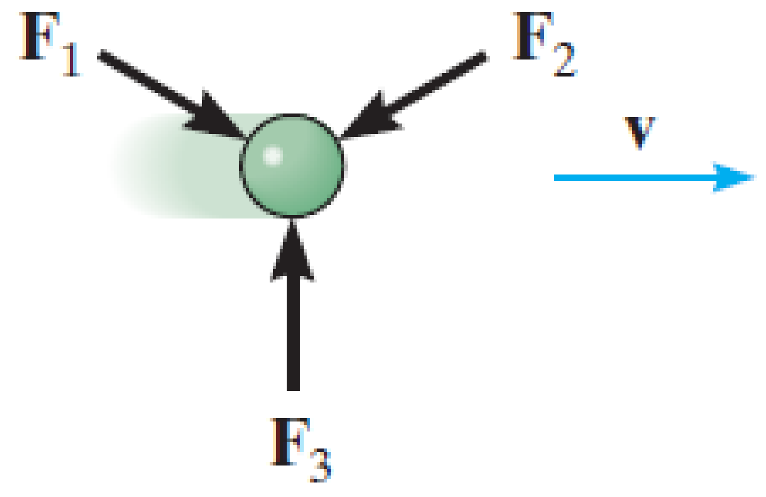
Newton’s first law

Newton’s first law of motion stats that if the resultant force system acting on a particle is zero, the particle will remain at rest or moving with a constant velocity .

This law provided the basic for the equation of equilibrium



Equilibrium

The resultant of each type of forces system was determined by obtained the sum of the forces of the system in certain directions and the sum of the moments of the forces with respect to certain axes. When all sums are zero for any particular force system, its resultant is zero and the body on which the system acts is in equilibrium.

Three main equation in equilibrium are:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

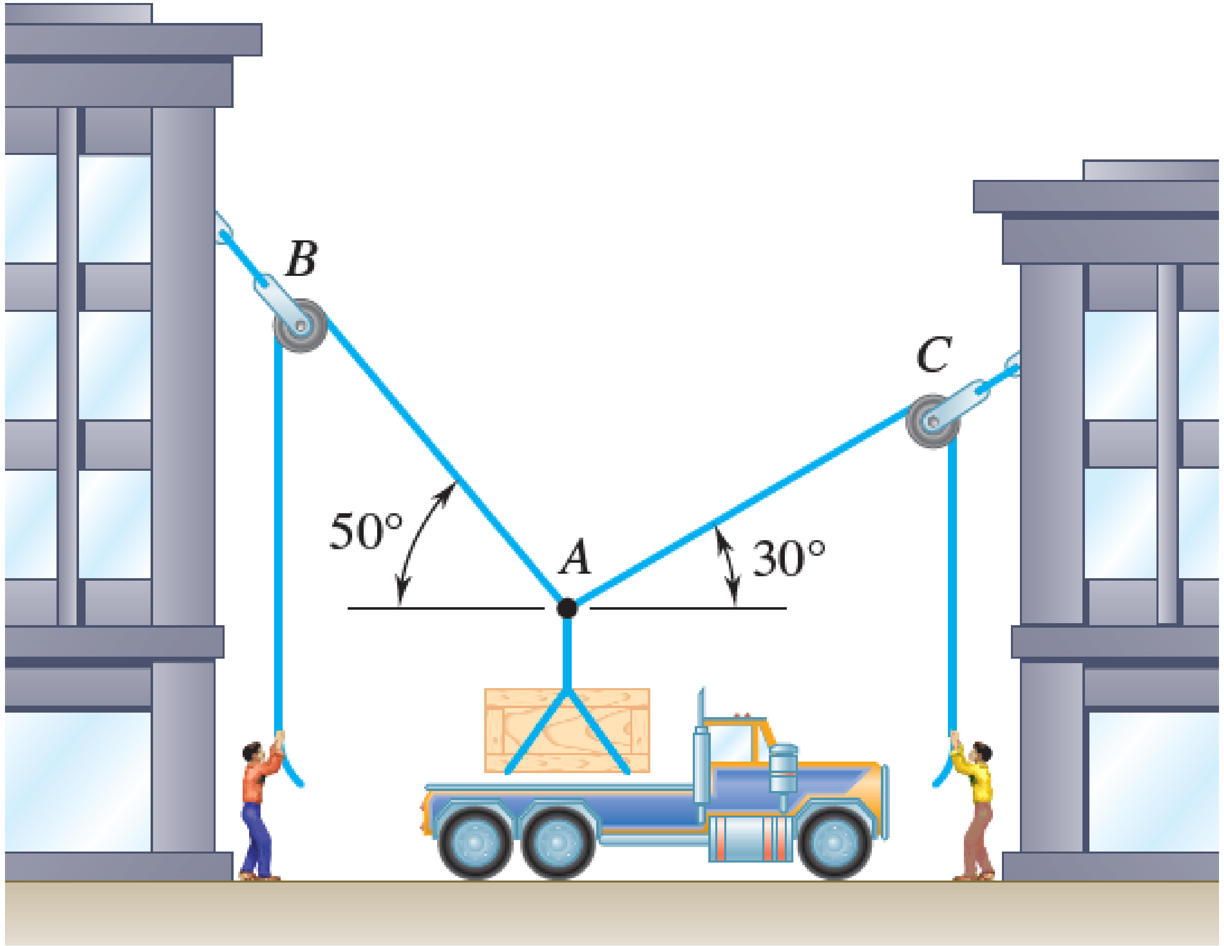
Free body diagram

A free body diagram is a sketch of a body, a portion of a body, or two or more bodies completely isolated or free from all other bodies, showing the forces exerted by all other bodies on the one being considered.

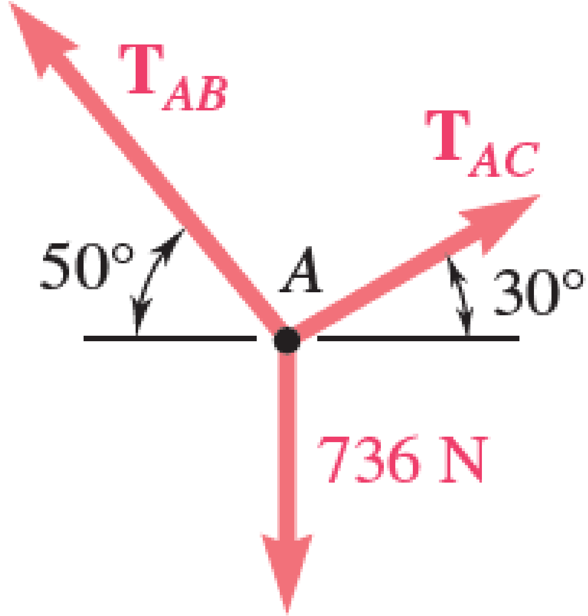
A free body diagram has three essential characteristics:

1. It's a diagram or sketch of the bodies.
2. The body is shown completely separated from all other bodies including foundations, supports and so on
3. The action on the body of each body removed in the isolated process is shown as a forces.

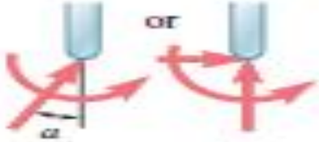
Free-Body Diagrams



(*a*) Space diagram



(*b*) Free-body diagram

Support or Connection	Reaction	Number of Unknowns
 Rollers Rocker Frictionless surface	 Force with known line of action perpendicular to surface	1
 Short cable Short link	 Force with known line of action along cable or link	1
 Collar on frictionless rod Frictionless pin in slot	 Force with known line of action perpendicular to rod or slot	1
 Frictionless pin or hinge Rough surface	 Force of unknown direction	2
 Fixed support	 Force and couple	3

This rocker bearing supports the weight of a bridge. The convex surface of the rocker allows the bridge to move slightly horizontally.

Links are often used to support suspended spans of highway bridges.

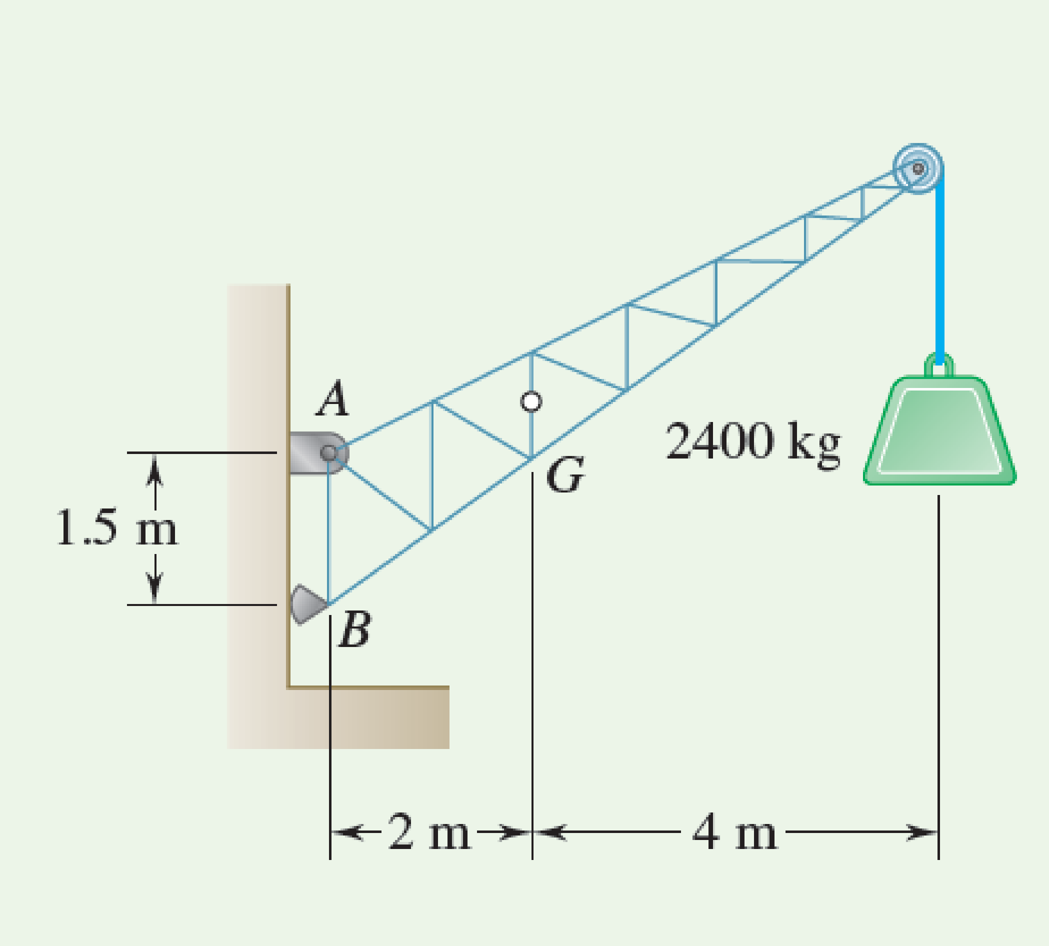
Force applied to the slider exerts a normal force on the rod, causing the window to open.

Pin supports are common on bridges and overpasses.

This cantilever support is fixed at one end and extends out into space at the other end.

A fixed crane has a mass of 1000 kg and is used to lift a 2400-kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*. Determine the components of the reactions at *A* and *B*.

STRATEGY: Draw a free-body diagram to show all of the forces acting on the crane, then use the equilibrium equations to calculate the values of the unknown forces.



Free-Body Diagram.

By multiplying the masses of the crane and of the crate by g = 9.81 m/s^2, you obtain the corresponding weights—that is, 9810 N or 9.81 kN, and 23 500 N or 23.5 kN (Fig. 1). The reaction at pin *A* is a force of unknown direction; you can represent it by components $\mathbf{A}_x$ and $\mathbf{A}_y$. The reaction at the rocker *B* is perpendicular to the rocker surface; thus, it is horizontal. Assume that $\mathbf{A}_x$, $\mathbf{A}_y$, and **B** act in the directions shown

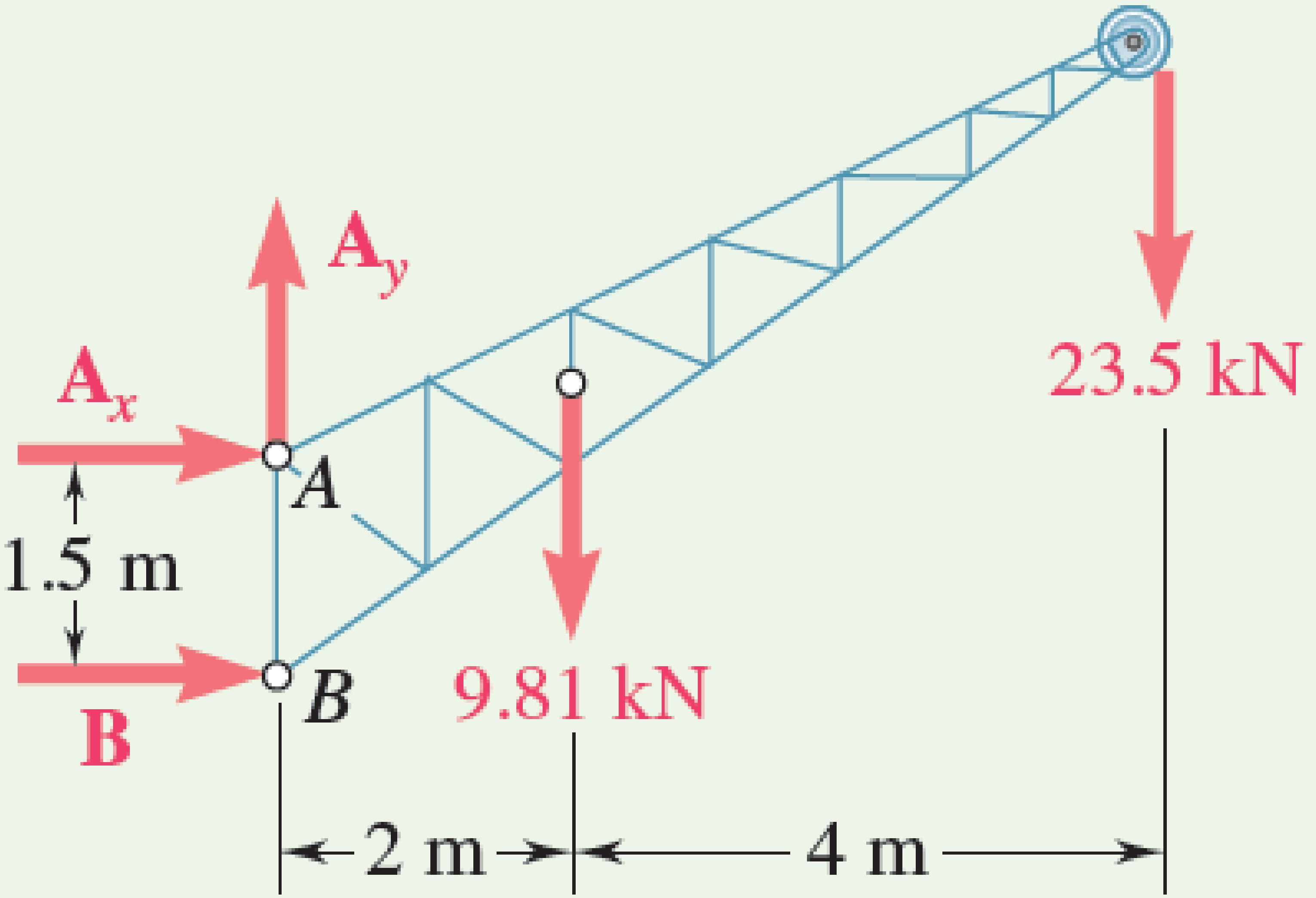


Fig. 1 Free-body diagram of crane.

Determination of B. The sum of the moments of all external forces about point A is zero. The equation for this sum contains neither A_x nor A_y, because the moments of $\mathbf{A}_x$ and $\mathbf{A}_y$ about A are zero. Multiplying the magnitude of each force by its perpendicular distance from A, you have

$$+\circlearrowleft\Sigma M_A = 0: \qquad +B(1.5\text{ m}) - (9.81\text{ kN})(2\text{ m}) - (23.5\text{ kN})(6\text{ m}) = 0$$
$$B = +107.1\text{ kN} \qquad \mathbf{B} = 107.1\text{ kN} \rightarrow \quad \blacktriangleleft$$

Because the result is positive, the reaction is directed as assumed.

Determination of $\mathbf{A}_x$. Determine the magnitude of $\mathbf{A}_x$ by setting the sum of the horizontal components of all external forces to zero.

$$\overset{+}{\rightarrow}\Sigma F_x = 0: \qquad A_x + B = 0$$
$$A_x + 107.1\text{ kN} = 0$$
$$A_x = -107.1\text{ kN} \qquad \mathbf{A}_x = 107.1\text{ kN} \leftarrow \quad \blacktriangleleft$$

Because the result is negative, the sense of $\mathbf{A}_x$ is opposite to that assumed originally.

Determination of $\mathbf{A}_y$. The sum of the vertical components must also equal zero. Therefore,

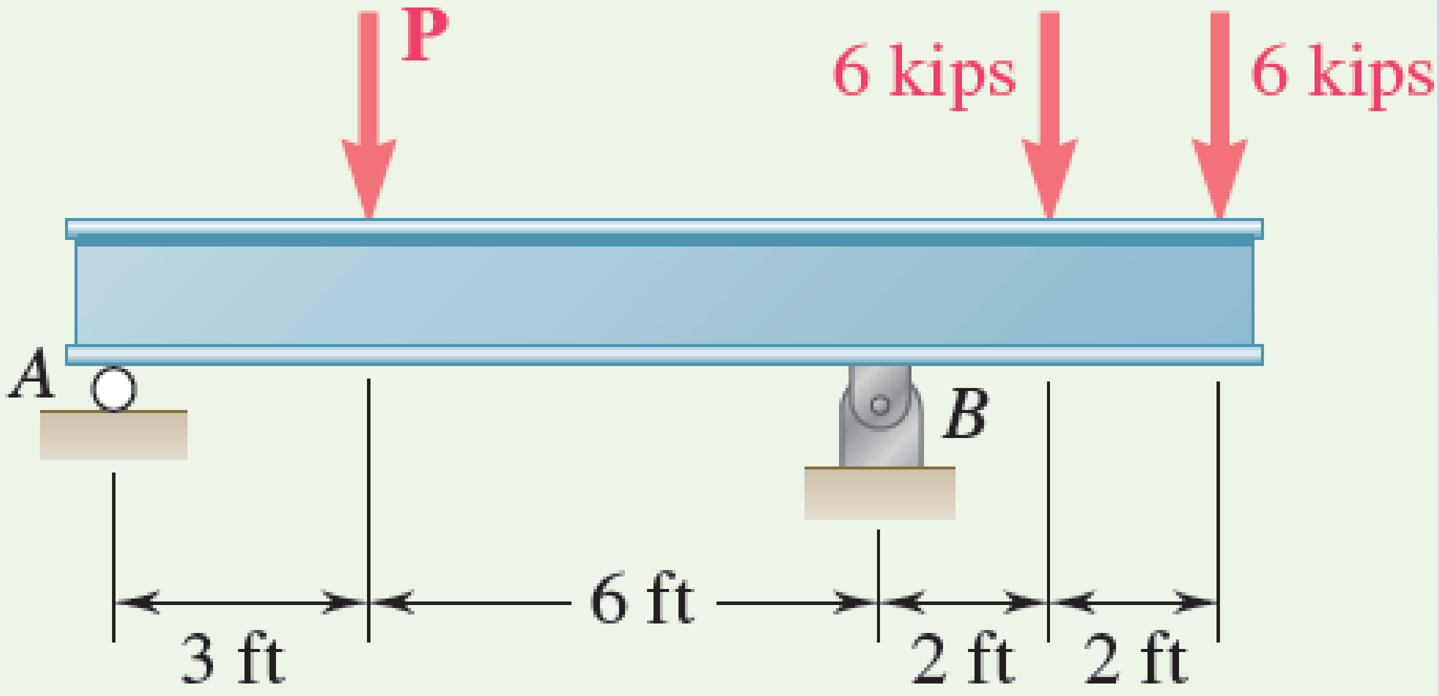
$$+\uparrow\Sigma F_y = 0: \qquad A_y - 9.81\text{ kN} - 23.5\text{ kN} = 0$$
$$A_y = +33.3\text{ kN} \qquad \mathbf{A}_y = 33.3\text{ kN}\uparrow \quad \blacktriangleleft$$

Adding the components $\mathbf{A}_x$ and $\mathbf{A}_y$ vectorially, you can find that the reaction at A is 112.2 kN $\measuredangle$17.3°.

REFLECT and THINK: You can check the values obtained for the reactions by recalling that the sum of the moments of all the external forces about any point must be zero. For example, considering point *B* (Fig. 2), you can show that

$$+\circlearrowleft \Sigma M_B = -(9.81 \text{ kN})(2 \text{ m}) - (23.5 \text{ kN})(6 \text{ m}) + (107.1 \text{ kN})(1.5 \text{ m}) = 0$$

Three loads are applied to a beam as shown. The beam is supported by a roller at *A* and by a pin at *B*. Neglecting the weight of the beam, determine the reactions at *A* and *B* when *P* = 15 kips.



Free-Body Diagram

. The reaction at *A* is vertical and is denoted by **A** (Fig. 1). Represent the reaction at *B* by components **B***x* and **B***y* . Assume that each component acts in the direction shown.

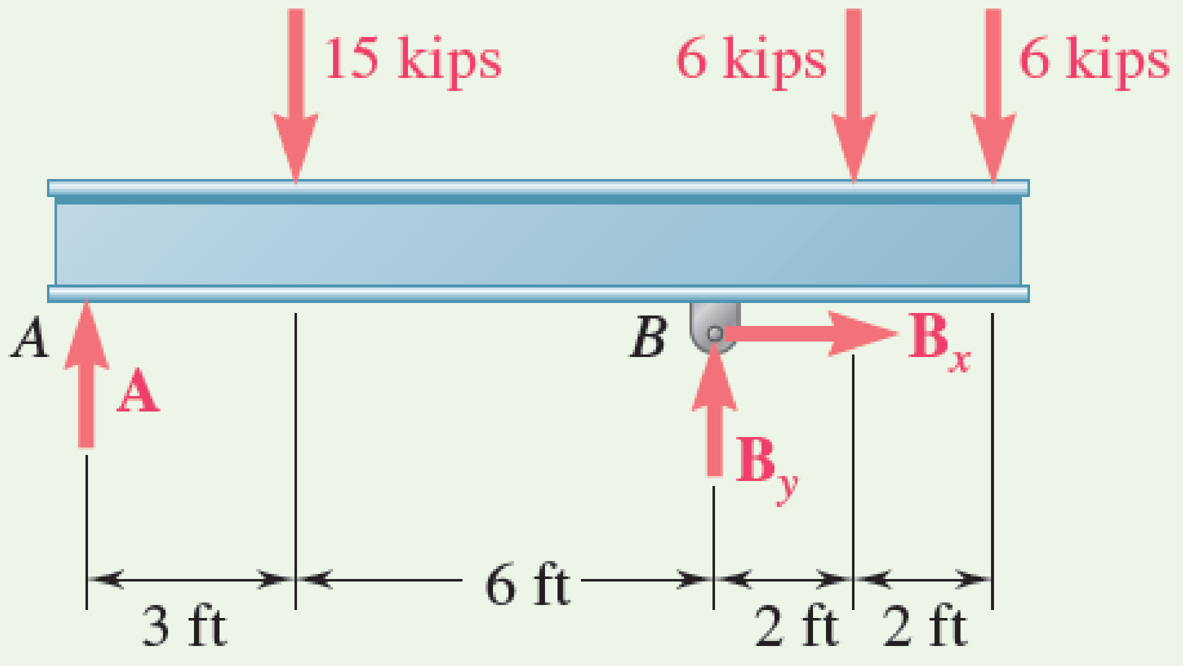


Fig. 1 Free-body diagram of beam.

Equilibrium Equations. Write the three equilibrium equations and solve for the reactions indicated:

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\quad B_x = 0 \qquad \mathbf{B}_x = 0$ ◀

$+\circlearrowleft\Sigma M_A = 0$:

$$-(15 \text{ kips})(3 \text{ ft}) + B_y(9 \text{ ft}) - (6 \text{ kips})(11 \text{ ft}) - (6 \text{ kips})(13 \text{ ft}) = 0$$

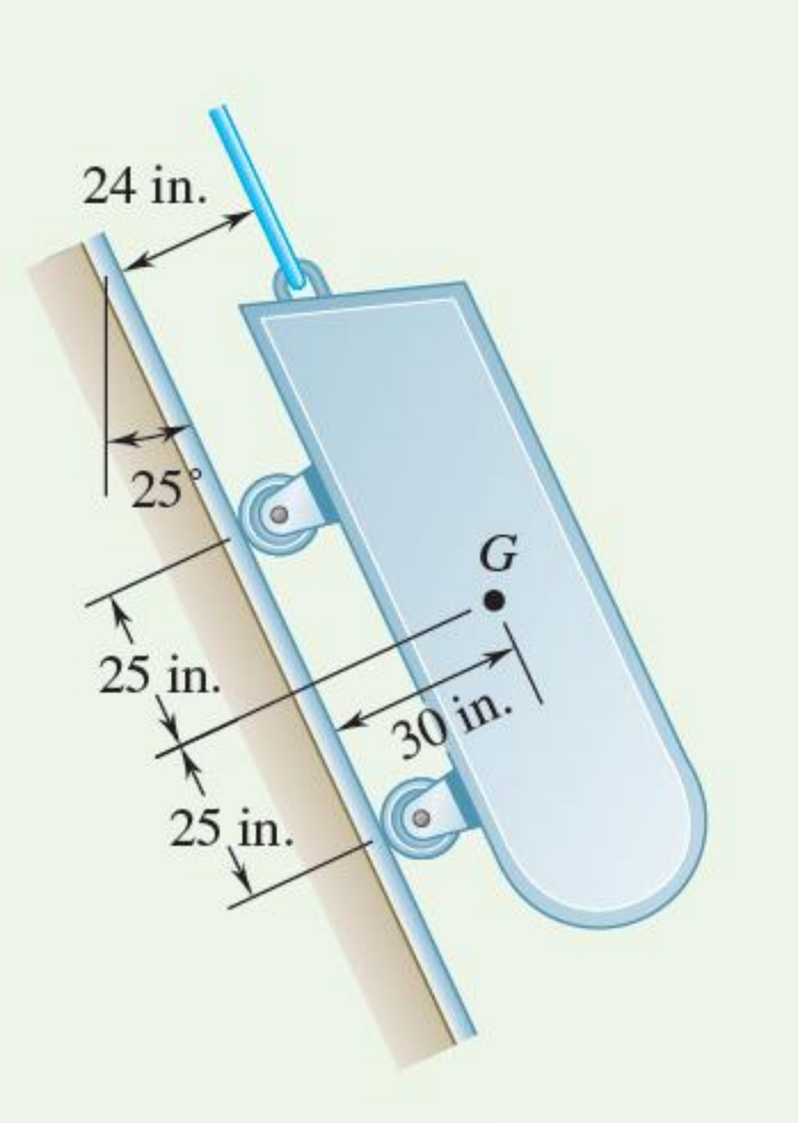
$$B_y = +21.0 \text{ kips} \qquad \mathbf{B}_y = 21.0 \text{ kips} \uparrow$$ ◀

$+\circlearrowleft\Sigma M_B = 0$:

$$-A(9 \text{ ft}) + (15 \text{ kips})(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (6 \text{ kips})(4 \text{ ft}) = 0$$

$$A = +6.00 \text{ kips} \qquad \mathbf{A} = 6.00 \text{ kips} \uparrow$$ ◀

A loading car is at rest on a track forming an angle of 25° with the vertical. The gross weight of the car and its load is 5500 lb, and it acts at a point 30 in. from the track, halfway between the two axles. The car is held by a cable attached 24 in. from the track. Determine the tension in the cable and the reaction at each pair of wheels.



Free-Body Diagram.

The reaction at each wheel is perpendicular to the track, and the tension force **T** is parallel to the track. Therefore, for convenience, choose the *x* axis parallel to the track and the *y* axis perpendicular to the track (Fig. 1). Then, resolve the 5500-lb weight into *x* and *y* components.

$W_x = +(5500 \text{ lb}) \cos 25° = +4980 \text{ lb}$

$W_y = -(5500 \text{ lb}) \sin 25° = -2320 \text{ lb}$

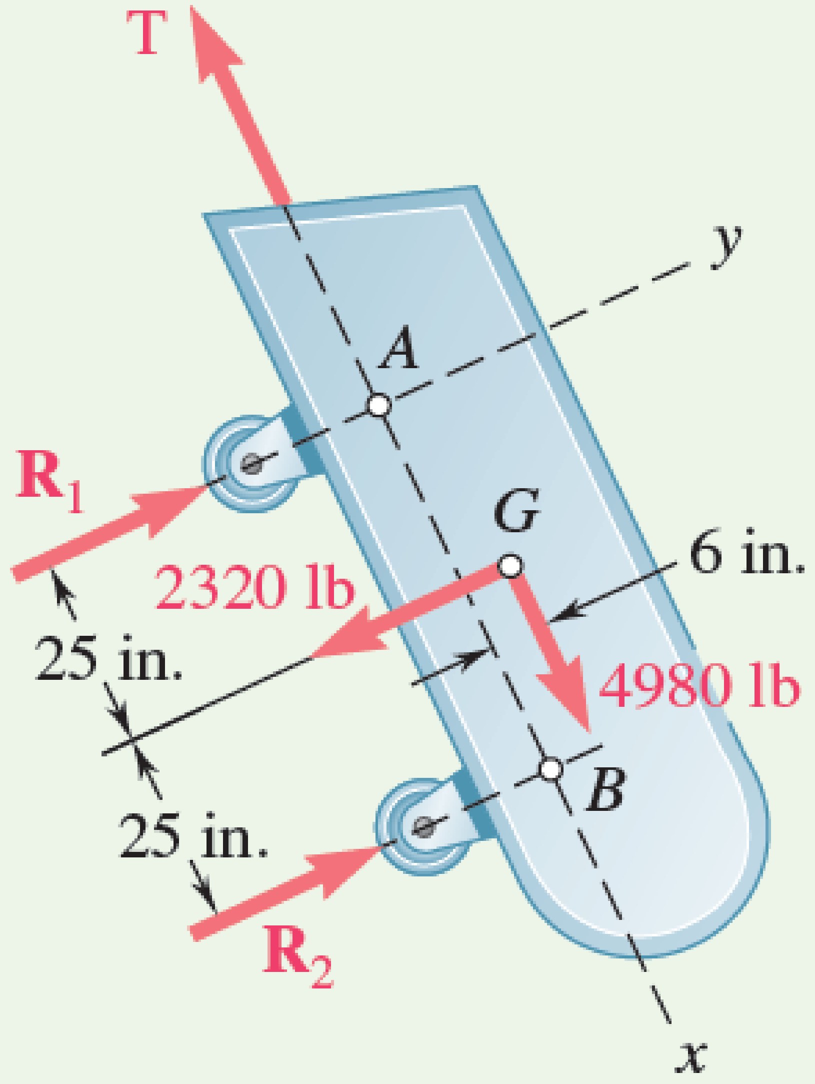


Fig. 1 Free-body diagram of car.

Equilibrium Equations. Take moments about A to eliminate $\mathbf{T}$ and $\mathbf{R}_1$ from the computation.

$$+\circlearrowleft \Sigma M_A = 0: \qquad -(2320 \text{ lb})(25 \text{ in.}) - (4980 \text{ lb})(6 \text{ in.}) + R_2(50 \text{ in.}) = 0$$

$$R_2 = +1758 \text{ lb} \qquad \mathbf{R}_2 = 1758 \text{ lb} \nearrow \quad \blacktriangleleft$$

Then, take moments about B to eliminate $\mathbf{T}$ and $\mathbf{R}_2$ from the computation.

$$+\circlearrowleft \Sigma M_B = 0: \qquad (2320 \text{ lb})(25 \text{ in.}) - (4980 \text{ lb})(6 \text{ in.}) - R_1(50 \text{ in.}) = 0$$

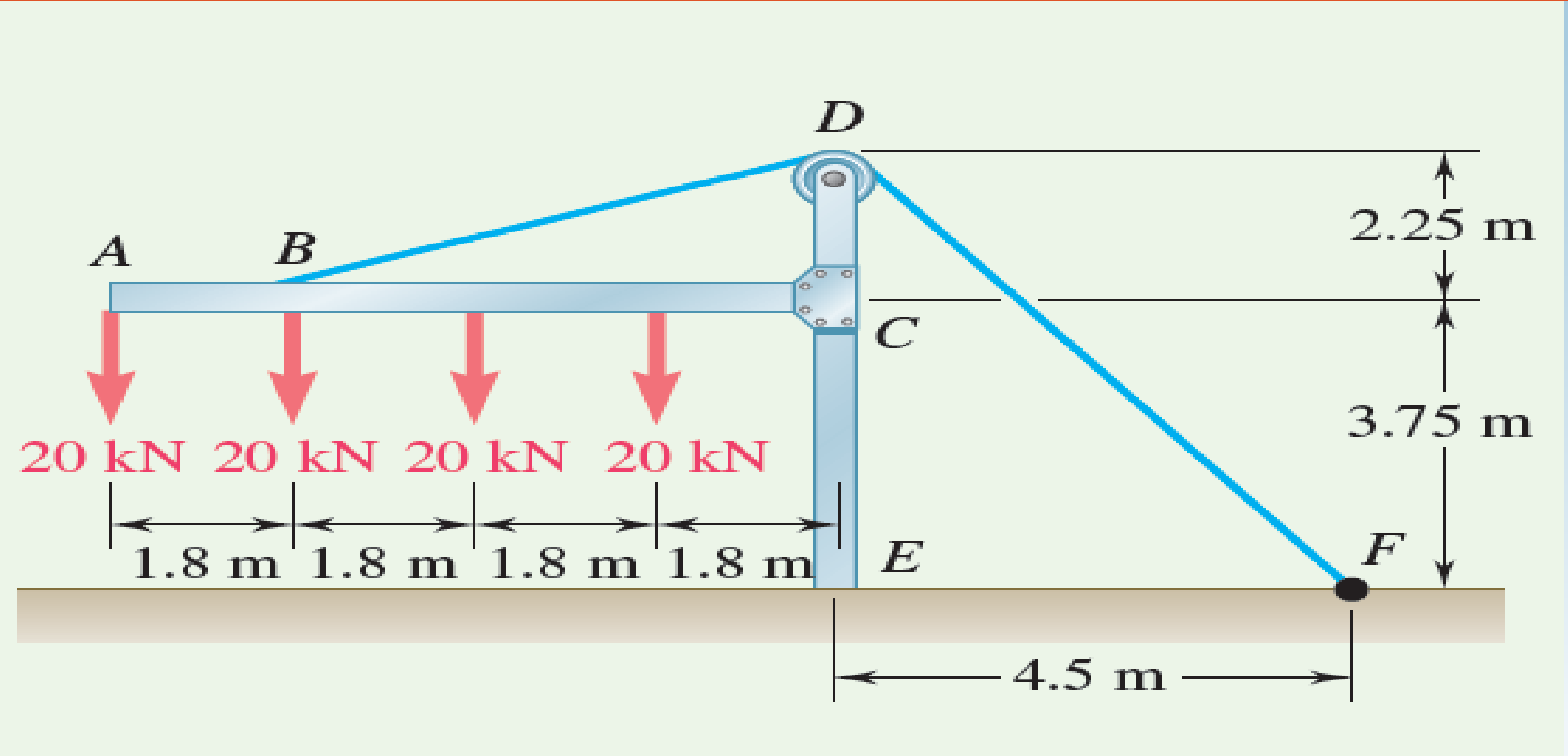
$$R_1 = +562 \text{ lb} \qquad \mathbf{R}_1 = +562 \text{ lb} \nearrow \quad \blacktriangleleft$$

Determine the value of T by summing forces in the x direction.

$$\searrow + \Sigma F_x = 0: \qquad +4980 \text{ lb} - T = 0$$

$$T = +4980 \text{ lb} \qquad \mathbf{T} = 4980 \text{ lb} \nwarrow \quad \blacktriangleleft$$

The frame shown supports part of the roof of a small building. Knowing that the tension in the cable is 150 kN, determine the reaction at the fixed end *E*.



Free-Body Diagram.

Represent the reaction at the fixed end *E* by the force components **E***x* and **E***y* and the couple **M***E* (Fig. 1). The other forces acting on the free body are the four 20-kN loads and the 150-kN force exerted at end *F* of the cable

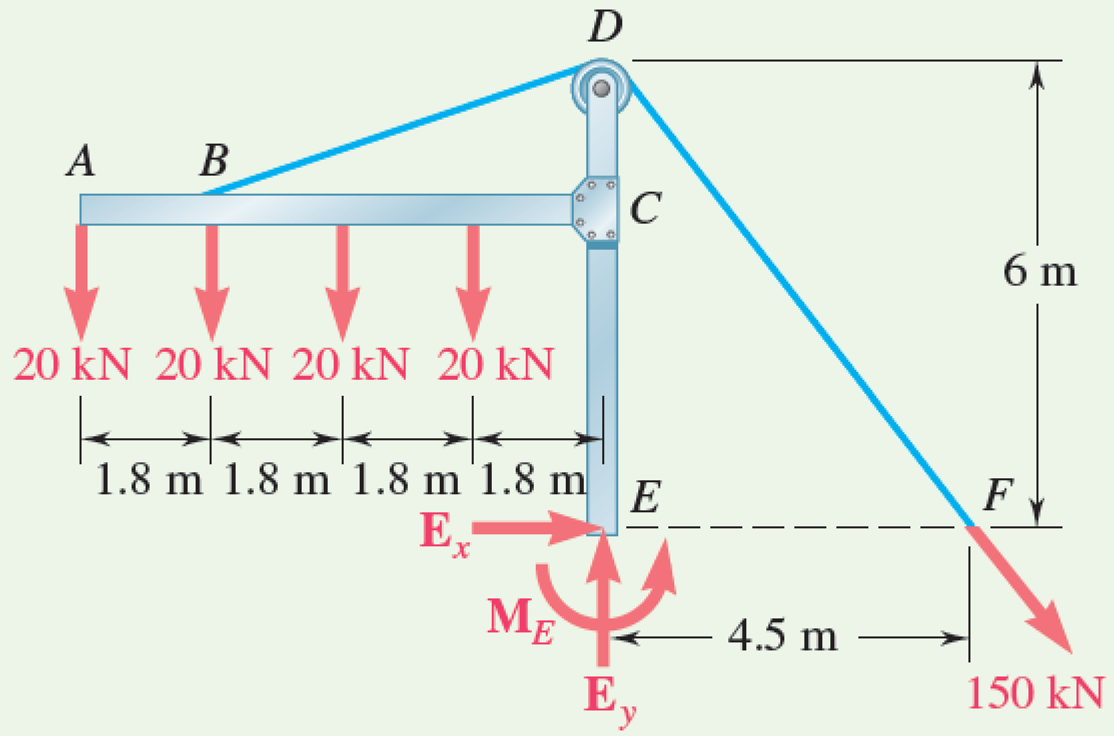


Fig. 1 Free-body diagram of frame.

Equilibrium Equations. First note that

$$DF = \sqrt{(4.5\ \text{m})^2 + (6\ \text{m})^2} = 7.5\ \text{m}$$

Then, you can write the three equilibrium equations and solve for the reactions at E.

$$\overset{+}{\rightarrow}\Sigma F_x = 0: \qquad E_x + \frac{4.5}{7.5}(150\ \text{kN}) = 0$$

$$E_x = -90.0\ \text{kN} \qquad \mathbf{E}_x = 90.0\ \text{kN} \leftarrow \quad ◄$$

$$+\uparrow\Sigma F_y = 0: \qquad E_y - 4(20\ \text{kN}) - \frac{6}{7.5}(150\ \text{kN}) = 0$$

$$E_y = +200\ \text{kN} \qquad \mathbf{E}_y = 200\ \text{kN} \uparrow \quad ◄$$

$$+\circlearrowleft\Sigma M_E = 0: \qquad (20\ \text{kN})(7.2\ \text{m}) + (20\ \text{kN})(5.4\ \text{m}) + (20\ \text{kN})(3.6\ \text{m})$$

$$+(20\ \text{kN})(1.8\ \text{m}) - \frac{6}{7.5}(150\ \text{kN})(4.5\ \text{m}) + M_E = 0$$

$$M_E = +180.0\ \text{kN}\cdot\text{m} \qquad \mathbf{M}_E = 180.0\ \text{kN}\cdot\text{m} \circlearrowleft \quad ◄$$

Thank you for listening